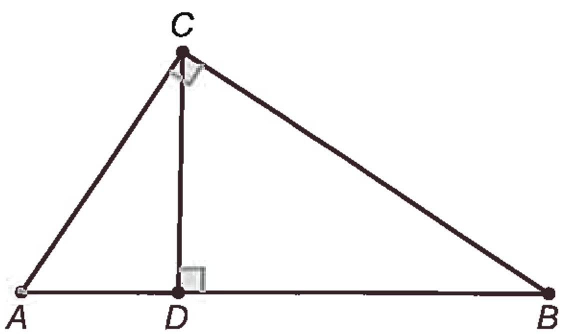
Geometry R – Mr. Bo
Unit 9 – Day 2 Notes

Name Notes.
Date________________

The Altitude of a Right Triangle

Altitude: A segment drawn from a vertex, perpendicular to the side opposite the vertex.



1. Identify and describe the three Altitudes of Right ΔABC.

$\overline{AC}$ is also a leg. of the Δ.

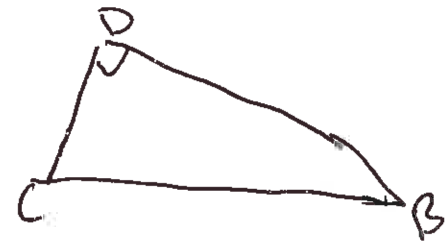
$\overline{BC}$ is also a leg of the Δ.

$\overline{CD}$ which is not also a leg.

Theorem: The *Altitude* drawn to the *Hypotenuse* of a right triangle splits the triangle into two smaller right triangles each *similar* to each other and to the original right triangle.

$\Delta ACB \sim \Delta ADC \sim \Delta CDB$.

2a. Complete the table using the fact that $\Delta ADC \sim \Delta CDB$.



Side of ΔADC	**Corresponding Side of ΔCDB**
$\overline{AD}$	$\overline{CD}$
$\overline{CD}$	$\overline{BD}$

b. Set up a proportion using the sides identified in the table above.

$$\frac{AD}{CD} = \frac{CD}{BD}$$

c. In part b, notice that Altitude $\overline{CD}$ shows up twice in the proportion, such that when you cross-multiply, CD gets multiplied times itself. We call CD the **Geometric Mean** of the proportion.

The Geometric Mean: x is the *Geometric Mean* of two values *a* and *b* if $\frac{a}{x} = \frac{x}{b}$.

3.

a. Find the geometric mean between 9 and 16.

$\frac{9}{x} = \frac{x}{16}$

$x^2 = 144$

$x = 12$

b. Find the geometric mean between 5 and 25

$\frac{5}{x} = \frac{x}{25}$

$x^2 = 125$

$x = \sqrt{125}$

$= 5\sqrt{5}$

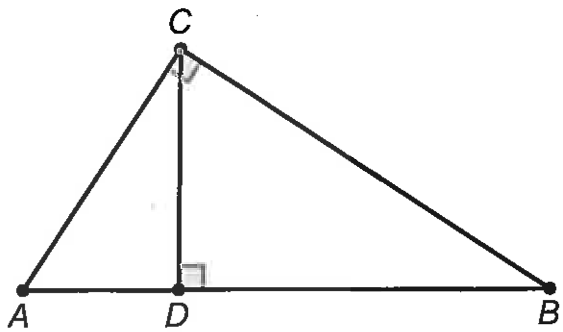
c. 8 is the geometric mean between 2 and what other number?

$\frac{2}{8} = \frac{8}{x}$

$2x = 64$

$x = 32$

The Altitude Rule:



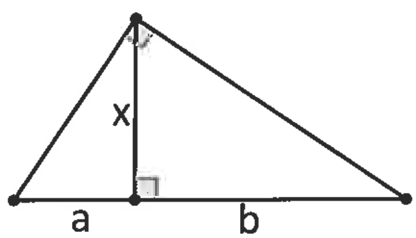
In 2b you set up the following proportion:

$$\frac{AD}{CD} = \frac{CD}{DB}$$

Notice that Altitude $\overline{CD}$ is splitting hypotenuse $\overline{AB}$ into 2 smaller segments: $\overline{AD}$ & $\overline{DB}$.

Furthermore, it was stated that CD is the **Geometric mean** of AD and DB.

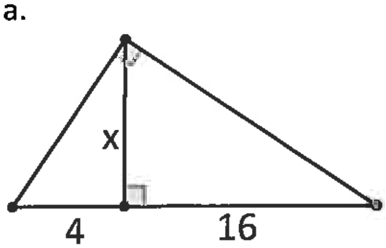
The Altitude Rule: The Altitude to the hypotenuse of a right triangle spits the hypotenuse into 2 parts such that it is the **Geometric Mean** between those parts.



part $\rightarrow$ $\frac{a}{x} = \frac{x}{b}$ $\leftarrow$ part.

4. The altitude to the hypotenuse of the right triangle is drawn. Solve for x.

a.

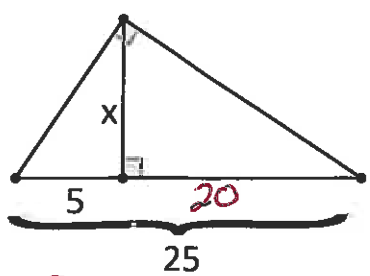


$\frac{4}{x} = \frac{x}{16}$

$64 = x^2$

$x = 8$

b.

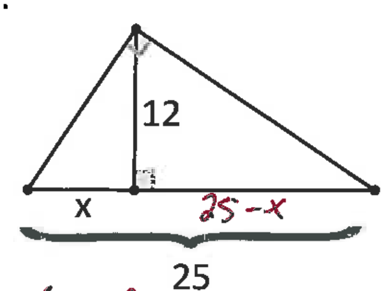


$\frac{5}{x} = \frac{x}{20}$

$x^2 = 100$

$x = 10$

c.



$\frac{x}{12} = \frac{12}{25-x}$

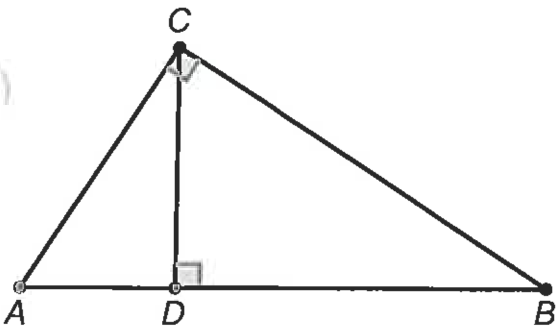
$144 = 25x - x^2$

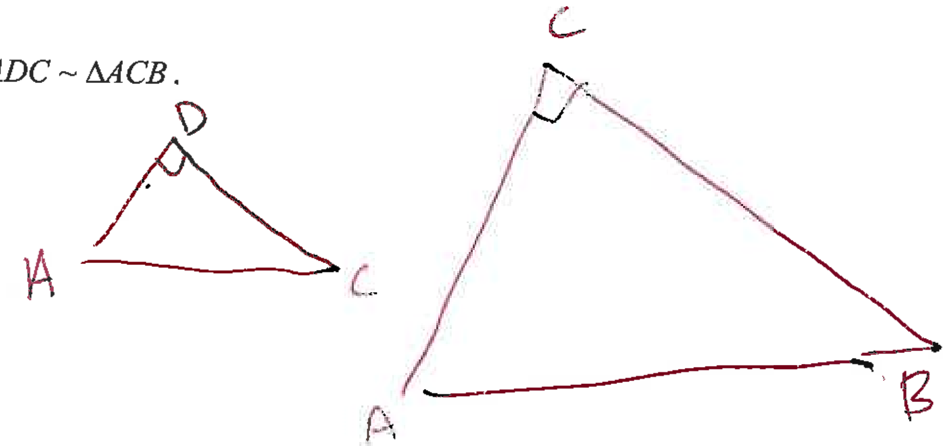
$x^2 - 25x + 144 = 0$

$(x-16)(x-9) = 0$

$x = 16$, $x = 9$

5a. Complete the table using the fact that $\Delta ADC \sim \Delta ACB$.





Side of ΔADC	Corresponding Side of ΔACB
$\overline{AD}$	$\overline{AC}$
$\overline{AC}$	$\overline{AB}$

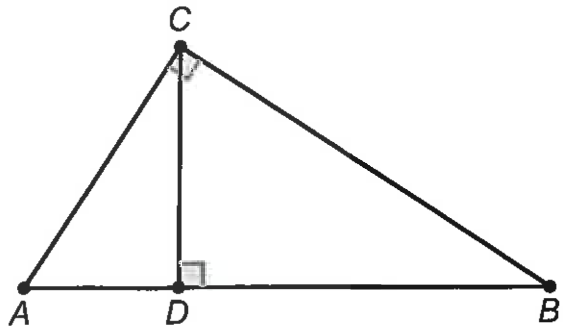
b. Set up a proportion using the sides identified in the table above.

$$\frac{AD}{AC} = \frac{AC}{AB}$$

c. Which value in your proportion is the **geometric mean**? How do you know?

AC (the leg of ΔACB) is the geometric mean because it shows up twice in the proportion.

The Leg Rule:



In 5b you set up the following proportion:

$$\frac{AD}{AC} = \frac{AC}{AB}$$

Notice that Altitude $\overline{CD}$ is splitting hypotenuse $\overline{AB}$ into 2 smaller segments: $\overline{AD}$ & $\overline{DB}$.

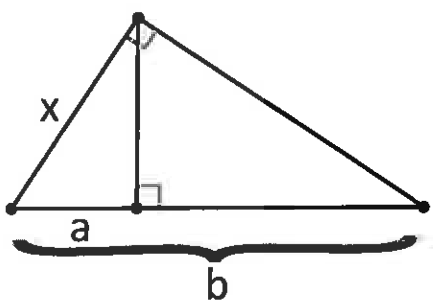
Furthermore, it was stated that AC is the **Geometric mean** of AD and AB.

The Leg Rule: The Altitude to the hypotenuse of a right triangle spits the hypotenuse into 2 parts such that the leg of the right triangle is the **Geometric Mean** between the hypotenuse and the part of the hypotenuse touching the leg.

part

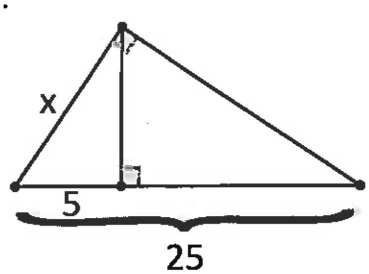
$$\frac{a}{x} = \frac{x}{b}$$

whole



6. The altitude to the hypotenuse of the right triangle is drawn. Solve for x.

a.



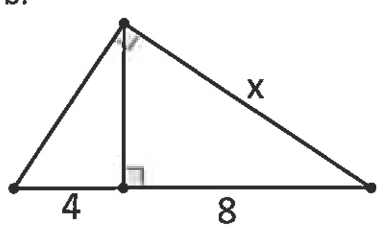
$$\frac{5}{x} = \frac{x}{25}$$

$$x^2 = 125$$

$$x = \sqrt{125}$$

$$= 5\sqrt{5}$$

b.



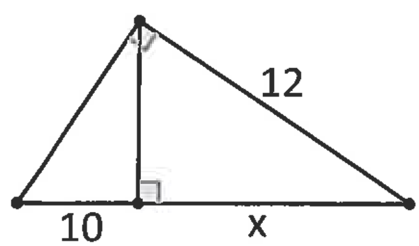
$$\frac{8}{x} = \frac{x}{12}$$

$$x^2 = 96$$

$$x = \sqrt{96}$$

$$= 4\sqrt{6}$$

c.



$10 + x$

$$\frac{x}{12} = \frac{12}{x+10}$$

$$x^2 + 10x = 144$$

$$x^2 + 10x - 144 = 0$$

$$(x+18)(x-8) = 0$$

$x = -18$, reject

$x = 8$